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FRACTALS. ORIGINS AND HISTORY

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Abstract: Fractal shapes can be found everywhere around us in Nature, at small scale microscopic level, at normal scale observable with the naked human eye, but also at very large scale level, when looking at very far distant galaxies. As it will be presented in this paper, there is an inherent relationship between fractal structures in Nature and the harmony perceived by human eye in the structures of Nature. Although not explicitly formulated until late XII century by Fibonacci, the very same Golden Proportion had been already approached by the ancient Greeks. This Divine Proportion which can be found around us in Nature, represents the base for the latter concept of fractal and is also the link between early mathematics and advanced fractal mathematics. Some of the first ideas about harmony and proportions are dating back in Antiquity and they were promoted by the ancient Greeks in their philosophical schools, by scholars like Euclid, Thales and Pythagoras. Although the concept of fractal it is a rather young one, being introduced by Mandelbrot in the '70s of the XXth century, the origins of the concept of fractal seem to be found, much earlier, in the aforementioned philosophical schools. In their philosophical schools, the antics, as far as we know, beginning with the Greeks, had discovered and used the concepts of proportion and harmony, in certain proportions they worked with. One such proportion is represented by the Golden Ratio.
Key words (TNR 10pt, bold, italic):fractal, fractal shapes, recursive rule, self-similarity, proportions, harmony, Golden Number

1. INTRODUCTION

Fractal shapes can be found everywhere around us in Nature, at small scale microscopic level, at normal scale observable with the naked human eye, but also at very large scale level, when looking at very far distant galaxies. A lot of scientific literature, such as in [4], [5] and [6], can be linked with the concept of fractal. As it will be presented in this paper, there is an inherent relationship between fractal structures in Nature and the harmony perceived by human eye in the structures of Nature. Although not explicitly formulated until late XII century by Fibonacci, as indicated in [1] and [2], the very same Golden Proportion had been already approached by the ancient Greeks. This Divine Proportion which can be found around us in Nature, represents the base for the latter concept of fractal and is also the link between early mathematics and advanced fractal mathematics. Some of the first ideas about harmony and proportions are dating back in Antiquity and they were promoted by the ancient Greeks in their philosophical schools by scholars like Euclid, Thales and Pitagora, according to [1] and [2]. Although the name of the concept of fractal it is rather young, being introduced by Mandelbrot in the '70s of the XXth century, the origins of the concept of fractal seem to be found very much earlier the aforementioned philosophical schools. In their philosophical schools, the antics, as far as we know, beginning with the Greeks, had discovered and used concepts like proportion and harmony, in some certain proportions they worked with. One such proportion is represented by the Golden Ratio, also extensively presented in [3].

It is well known that they were alreadytrying since then to find meaning and order inNature and also to explain relationships between measuresof different geometry objects. Although modern mathematics was not yet discovered at that time, the antiques were very familiar with concepts such as symmetry and self-similarity and recursive rules. Since many ancient writings and works did not withstand the time, there is little evidence about concerns of the antiques with this concept of fractal, as it will be named later. On the other side, monuments, constructions and art from ancient era are proof of the fact that antiques were rather familiar with geometric shapes, but also with the concept of proportionality and harmony, since one of the most famous theorems of geometry is attributed to the Greek Thales of Millet. These concepts of proportionality and harmony, initially discovered, understood and used by the Greek civilization were almost surely transmitted and also taken to the next level by Arabs and Indians in the Early Middle Ages. Mathematical knowledge has been thus improved and completed with new concepts by Arabs, and then further transmitted to the European civilization. During the Renaissance, mathematics had already advanced enough and scientists of the late Middle Ages were already more familiar with the not yet named concept of “fractal”.

2. ORIGINS OF THE CONCEPT OF FRACTAL IN ANTIQUITY

One of the main scientific personalities of Antiquity is Pythagoras (569/475 b.C.). He is considered to be the first one who dealt with the proportion related to the Golden Ratio, proportion which occurs few times in their specific pentagram symbol. The aforementioned proportion is then used by the Greek architect Phidias in some of his works and going out from the name of Phidias, the mathematician Mark Barr later also named this proportion or ration with the Greek letter Φ.

It is supposed that the work of Pythagoras has been transmitted to Euclid by his followers, probably Theodor Cyrene or even Plato.

Another main scientific personality of Antiquity is Euclid of Alexandria (326/265 b.C.), who conceived his work “Elements” as a text for the discipline of geometry, for the ancient University of Alexandria. Even to today teaching standards, this work of Euclid is a remarkable one. Therein, going out from certain intuitive axioms and postulates, Euclid constructs step by step an entire edifice for mathematics.

Fractals are about self-similarity mirrored in certain fixed proportions. It is very close at hand to suppose that, due to their obsession for proportions, antiques also had knowledge about fractal shapes which they encountered in Nature or which they themselves imagined, like the pentagram symbol of the Pythagoras School. Other proofs for this are of course very elusive, on one side because the knowledge shared in the philosophical schools was known as being secret and on the other side, many works had not had the luck of the work of Euclid the Elements, to withstand the period of a few millenniums.

3. ORIGINS OF THE CONCEPT OF FRACTAL IN MIDDLE AGE

The idea of Golden Ratio or Divine Proportion has been probably transmitted to Euclid by means of the writings of Theodor Cyrene or Plato. Euclid mentions this proportion in the sixth book of his text, The Elements. In the Early Middle Ages, the mathematical science of Indians and Arabs was far more advanced than mathematics of Europeans. At that time, Arabs had already developed the first astrolabes used to determine the positions of stars in the sky, very useful in maritime navigation. Indians had already recorded important contributions by using the present day numbering system and they also developed advanced algebraic procedures to calculate square roots.

The mathematical knowledge of Arabs and Indians was introduced in Europe only in the XII century, by Leonardo da Pisa (1170/1240 a.D.), who was also named Fibonacci. He was raised by his family in North Africa, since his father had worked in Algeria, as representative of the merchants from Pisa. Around 1200 a.D. Leonardo returns to Pisa and shares the knowledge he had acquired in Africa, with its European contemporaries. He authored many books, out of which only five are still kept. One of them is Liber Abaci, a book designated to those interested in mathematics. In the same book he also presents the numbering system of the Indians which will be adopted a little bit later by Europeans and also presents the Rabbit problem. This latter problem contains the so named Fibonacci sequence which is directly connected with the Golden Number or Divine Proportion. The work of Leonardo da Pisa is then transmitted to its successors like Luca Pacioli and Leonardo da Vinci, both of them very passionate about arithmetic and geometry, who both shared a serious interest for proportions. In his works, Pacioli recognizes the contributions of his predecessors, Fibonacci also included. Some four hundred years after Fibonacci, the Golden Number is still unknown but at that time the mathematician Albert Girard puts the Rabbit problem in its mathematical form, namely each term of the sequence is defined by summing the previous two members of the sequence. Only in 1753, Robert Simson remarks that the sequence obtained by dividing two consecutive terms of the Fibonacci sequence is convergent, the limit of this sequence actually being the Golden Number. Taking into account all what has been presented in this subchapter, it is rather obvious that in the Middle Ages, scholars like Luca Pacioli which is also the author of the work De DivinaProportione, had already understood the significance of the Golden Number in the Creation and Nature. By studying living and non-living Nature, they almost surely saw this Golden Proportion which also characterizes various fractals, around them in Nature. The harmony perceived by the most sensitive of the Middle Ages’ artists also contributed to the magic of this number.

4. MORE ABOUT FRACTALS

Although an exact and generally accepted and recognized definition of fractals does not exist yet, the main ideas used to define fractals are self-similarity, being based on recursiverules and they relate with the space they are embedded in. According to [8], a fractal is defined as “a rough or fragmented geometric shape which can be split into parts, each of which is a reduced size copy of the whole”.One point agreed upon is the concept of fractal dimension, which are numbers quantifying the complexity of a fractal. As already mentioned before, fractals are about self-similarity mirrored in certain fixed proportions. Self-similarity can be defined as symmetry across different scale. Almost all fractals exhibit at least partially self-similarity.In the Middle Ages, scholars became more and more obsessed

with the Golden Ratio and they tried to find this number everywhere around them. Even though the respective Divine Proportion cannot be found in Nature such often as originally believed, the obsession ofMiddle Ages mathematicians has seriously contributed to advances related to proportions and fractals, on the realm of mathematics. The occurrence of the Golden Section is mainly observable in arts, design and architecture and the explanation for this lays in the fact that it was perceived by the human eye as a very harmonic proportion. This perceived harmony is also the reason whyLe Corbusier introduced it in the standards of design in construction of buildings. In XVIIth century, G. Leibniz approached in his work the concept of recursive self-similarity, after the notion of recursion has been previously defined and used and thus, fractals were increasingly, more rigorously treated.

Fractal geometry is an extension or generalization of Euclidean geometry and is used to describe a great variety of self-similar objects. Fractals model complex physical processes but also dynamical systems. A main idea about fractals is that a simple rule or process, going through a lot of iterations becomes a very complex shape, structure or process.Since fractals are very simple processes which by many iterations produce very complicated results, this fact can be linked with the Chaos Theory with the following idea: if something represents a complicate result, does not necessarily mean that the initial input was complicated too.

In the work of B. Bolzano, B. Riemann and K. Weierstrass, in the XIXth century they were identified as continuous but non differentiable functions. After that, G. Cantor, who attended lectures of Weiestrass published a work containing examples of subsets, known as Cantor sets, with unusual properties, later recognized as fractals. At the end of XIXth century, F. Klein and H. Poincare introduced the so called “self-inverse” fractals. At the dawn of the XXth century, H. v. Koch extended the ideas of Poincare, by giving a more geometric definition and also producing hand drawn images of the so called Koch snowflake. A little bit later W. Sierpinski constructed his famous triangle gasket. P. Fatou and G. Julia, French mathematicians, came at results describing fractal behaviour associated with mapping complex numbers. Three years thereafter, F. Hausdorff seriously extended the definition of “dimension” to non-integer numbers. Only with manual drawings, without the aid of computers the knowledge about fractals was rather limited. That changed in 1960s, when B. Mandelbrot used computer simulation to better visualize fractals and also promoted the so called Mandelbrot sets. The same mathematician, Benoit Mandelbrot coined the term of “fractal” in 1975, a word based on the Latin word “fractus” which means “broken” or “fractured”. A bit later, in 1980, L. Carpenter introduced a software for creating fractal landscapes.

One important idearegarding fractals, dating back in 1965 was expressed by B. Mandelbrot and R. Feynman in [7], and regards the potential relationship between quantum phenomena, chaos theory and also processes generated by fractals, going through an infinity of iterations, based on the stochastic behaviour of all these mentioned phenomena. This idea will be later retaken and closer analysed.

One important key number which measure the degree of complexity of a fractal is the fractal dimension. The definition of fractal dimension is related to the concept of power law distribution. Data obeys a power law distribution if it fits the following equation:

$$y = cx^d \qquad (1)$$

If one applies the logarithm on the previous equation (1), then the respective relationship becomes linear, and can be rewritten in the following form:

$$log(y) = log(c) + dlog(x) \quad (2)$$

which represents a linear dependence between log(y) and log(x). The conclusion is that, if the plot of log(y) versus log(x) can be represented by a straight line, then the relationship between y and x represents a power law relationship with the slope d of the straight line plotted.

If one denotes with n, the number of pieces and with s the reduction factor, then the dimension (D) of a fractal can now be introduced with the following formula:

$$D = log_s\left(\frac{1}{n}\right) \quad (3)$$

For example, if a square is broken into four pieces by cutting each of its sides in half, then the dimension of this fractal is $D = log_{\frac{1}{2}}\left(\frac{1}{4}\right)$,that is to say D=2.

Considering now a Sierpinski Gasket, whereas the initial triangle is broken into three and the sides of this triangle are cut in half, then the dimension of this fractal is $D = log_{\frac{1}{2}}\left(\frac{1}{3}\right)$ = 1.585.

5. FRACTALS AND FRACTAL FEATURES IN NATURE AND NATURAL PHENOMENA

In nature, approximate fractals can be found everywhere around. Some phenomena including fractal features are: leaves of trees and algae, animal coloration patterns, blood and pulmonary vessels, clouds and rainfalls areas, coastlines craters, crystals, DNA, earthquakes, heart rates, lake or sea shorelines and areas, lightning bolts, networks, polymers, mountain ranges, ocean waves, proteins, river networks, snowflakes, soil pores, trees and dust grains, as stated in [8].

A branch of physical cosmology, called fractal cosmology is a set of theories which state that the distribution of matter in Universe or the structure of the universe itself, is a fractal. A central issue related to this interpretation of the Universe is the fractal dimension of the universe or of matter distribution within it, when measured at very large or very small scales.

First fractal model for the distribution of galaxies was generated in 1987 by L. Pietronero and his team. When the number of discovered galaxies grew larger, this initial model was later consequently completed. Furthermore, by admitting the Big Bang and Inflationary Universe Theories, we are dealing in this case of the inflationary universe with a fractal in dynamic or with a fractal dynamic at very large scale. This dynamics of shaping based on evolving fractal shapes in time are amain goal of the fractal theory, because most natural processes and phenomena are dynamic processes evolving in time.

Fractals can be also found in a wide range of applications in technology, such as: fractal antennas, fractal transistors, fractal heat exchangers, digital imaging, architecture and urban growth, fractal landscape or coastline complexity, enzymes, signal and image compression, computer video design and computer graphics, procedural generation, fracture mechanics, technical analysis of price series, networks, medicine, neuroscience, diagnostic imaging, pathology, geology, geography, archaeology and seismology. Very important for discovering and understanding fractal dynamics is, as it will be later shown, the procedural generation based on certain specific matrices used to identify and detect hidden rules of natural living or non livingfractal structures and shapes.

6. CONCLUSIONS

Fractal shapes and structures can be found everywhere around us in nature, but also in biological structures of our own human bodies. The roots of the concept of fractal and the initially related knowledge are to be found in the ancient philosophical schools of Greeks, knowledge which has been then transmitted to Arab and Indian civilizations. These origins returned to Europe only in the XII century by means of Leonardo da Pisa (Fibonacci) and they subsequently developed during whole Middle Ages. These origins of the concept of fractal can actually be found in the approaches of proportions by antiques, whose ideas were later retaken, improved and completed by scholars of the Middle Ages. The scientific personalities of the Middle Ages were people with multiple scientific and artistic background, for example, such as Luca Pacioli was, with contributions in arithmetic, geometry, art, architecture and accounting. Their multi- and interdisciplinary approach was very fruitful in linking the existent knowledge at that time and in gaining a much more deeper understanding of science, in general.The fascination produced by the Golden Ratio and the harmony perceived therein in the Middle Ages, has stimulated and hasseriously helped important advances in mathematics.The term “fractal” was coined in the ‘70s of the XXth century by B. Mandelbrot. The subsequent large availability of computing power further helped to understand fractals and to generate fractal shapes, which previously were impossible to be generated only by hand drawings.

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