Processing in (linear) systems with stochastic input

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ABSTRACT

The paper is providing a different approach to real-world systems, such as micro and macro systems of our real life, where the man has little or no influence on the system, either not knowing the rules of the respective system or not knowing the input of the system, being thus mainly only spectator of the system's output.
In such a system, the input of the system and the laws ruling the system could be only "guessed", based on intuition or previous knowledge of the analyzer of the respective system.
But, as we will see in the paper, it exists also another, more theoretical and hence scientific way to approach the matter of the real-world systems, and this approach is mostly based on the theory related to Schrödinger's equation and the wave function associated with it and quantum mechanics as well.
The main results of the paper are regarding the utilization of the Schrödinger's equation and related theory but also of the Quantum mechanics, in modeling real-life and real-world systems.

Keywords: (linear) system, transfer function, probability wave function, Schrödinger's equation, quantum mechanics, Hilbert space

1. INTRODUCTION

The Theory of Systems is a very intricate one, aiming to explain the functioning of the systems, systems ranging from the simplest ones, which are namely the technical systems created by man, such as automation systems and electronic systems up to the most complex systems of the nature, being micro systems or macro systems of the real world.

In the more simple technical systems created by man, the input, the state and the output of the system are mainly known and mostly controlled by man. The present paper deals especially with those systems where the man has little or no influence on the system, either not knowing the rules of the respective system or not knowing the input of the system, being thus mainly only spectator of the output of the system.

In the present paper considerations about these real-world (linear) systems will be made with the purpose to gain some additional knowledge on this kind of systems. The assumption of linearity of these systems, although a restrictive one and not always a valid one for real-life systems, is at present, one of the best theoretical assumptions we can use within the present state of knowledge.

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1. (LINEAR) SYSTEMS, TRANSFER FUNCTION AND THEORY

To the general form of a linear system[1],[2] is corresponding the diagram from the following Figure1:

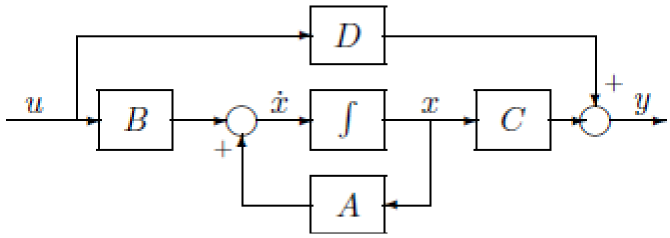

Figure 1. Diagram of a linear system

The set of equations which are describing a linear system with the abovementioned diagram, are:

$$\begin{cases} x(t) = A\dot{x(t)} + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \qquad (1)$$

where A, B, C and D are representing matrices describing the rules of the linear system, whose dimensions are corresponding to the relations between them.

X is representing the state of the linear system, U is representing the input of the linear system and Y is representing the output of the system.

Taking into account some initial conditions imposed to the above system, such as stated in (3), the transfer function of the aforementioned linear system can be calculated as:

$$T(s) = C(sI - A)^{(-1)}B + D \qquad (2)$$

It is obvious that the transfer matrix T(s) is a (m, n) matrix, where n is representing the dimension of the vector U(s). Then obviously, it follows: Y(s)=T(s)U(s)

Nor insisting upon the derivation of the transfer function of a linear system and neither on the theory related to the connection of the systems, the main possible connections of the linear systems are the serial and parallel connection, with the resulting transfer functions:

$$T_s = T_n T_{n-1} \dots T_1 \qquad (3)$$

and

$$T_p = T_1 + T_2 + \cdots + T_n \qquad (4)$$

Reverting now to the quantities X, U, Y and T, we are making the following considerations on them:

X, representing the state of the linear system is almost always a observable and determinable quantity of a system, at a certain moment in time.

Y, representing the output of the system is also almost always observable and determinable quantity in the system.

T is representing the transfer function of the system. Unfortunately we usually do not know the exact expression of the function T, but we do know however its approximation from past observations of the system.

U, representing the input of the system is an almost never observable nor determinable quantity, mainly because of the fact that is representing extern factors of the system which could affect or not affect the real world system, and moreover, even if assuming that we could know this U, we do not possess the senses to determine which components of the U will interact and affect the system.

Although we do not know U, we may logically assume that this is a non-deterministic function, a purely stochastic quantity or a probability function referring and sometimes pertaining to the system itself.

As already mentioned in the Abstract, the most important assumption of the paper, is to associate with this purely stochastic input function U, a wave function, such as the wave function from the Schrödinger's equation is. This wave function, which is usually used for quantum particles and noted with Ψ(x,t), is a function dependent on the position x of the particle at a certain moment of time t, which is describing the phenomena down at quantum level.

The Ψ wave function[6],[7],[8],[9],[10],[11] and hence the form of the Schrödinger's equation is more simple for the case of a single particle, but the wave function and the form of the Schrödinger's equation are more complicated for the case of many interacting particles, in the form of the Schrödinger's equation occurring thus additionally constraints, represented by the additional potential function V.

The wave function corresponding to a single particle without any constraints, has the form:

$$\Psi(x,t) = Ae^{i(kx-\omega t)} \qquad (5),$$

where A can be a real or complex number.

2. SYSTEMS WITH SCHRÖDINGER'S WAVE FUNCTION INPUT

As stated before, for the stochastic input, it is taken into consideration the Schrödinger's wave function[6],[7],[8],[9],[10],[11]. The state of the result (output) which is following the wave function input is actually a quantum state, and it is described by the density matrix that is in this case a number (ρ), characterizing either a mixed state of the quantum system, a statistical ensemble or several quantum states.

The numbers p_i are probabilities, that is to say they are fulfilling the conditions:

$$0 \leq p_i \leq 1 \qquad (6)$$

and

$$\sum_i p_i = 1 \qquad (7)$$

If p_i is the probability that the system is in the state Ψ_i, it can be considered, than the number (ρ_i), characterizing the Ψ-state of the system is given by the formula:

$$\rho_i = \sum_i p_i \hat{\Psi}_i \qquad (8)$$

If p_j is the probability that the system is in the state Φ_j, it can be considered, than the number (ρ_j), characterizing the Φ-state of the system is given by the formula:

$$\rho_j = \sum_j p_j \hat{\Phi}_j \qquad (9)$$

The probability that the system is passing (collapsing) from the Ψ-state into the Φ-state, it can be calculated using the inner product associated with Hilbert space[3], [4] and [5], namely:

$$P(\Psi \rightarrow \Phi) = \|\langle \Psi, \Phi \rangle\|^2 \qquad (10)$$

where Ψ and Φ are eigenvectors.

Hence the output of the system could be found in the very same state of the system, characterized by the same density (ρ_1) or it could be found in a different state, characterized by another density (ρ_2), the probability for that collapsing being calculated with the formula above.

3. POSSIBLE SITUATIONS FOR THE COMBINATION OF STOCHASTIC INPUT FUNCTION (WAVE FUNCTION), Ψ(x,t) STATE OF SYSTEM AND THE TRANSFER FUNCTION, T

The present section of the paper is cathegorizing the situations in which a certain system could be found, depending on the main attributes and components of the system: the input, transfer function, output and state of the system.

Taking into consideration the relationship between the output (Y) and the input (U or Ψ), we assume that, in order to have the output of the system in the same state, the input should be represented by the same wave function (Ψ).

It is also close at hand to conclude that different input wave functions (Us or Ψs), are generating outputs that are not in the same state.

Therefore in accordance with those above observations, the possible situations for the combination of the input wave function (U or Ψ), state of the system, transfer function (T) and output (Y) are:

a) Systems with the same input wave function U, same state of output Y and same transfer function T. For this kind of systems the output of the systems after the n^{th} processing is described by the relation:

$$Y_n = T^n U \qquad (11)$$

b) Systems with the same input wave function U, same state of output Y and different transfer functions (T_i) for each processing in the system. Hence the relation for the output of the system after the n^{th} processing is:

$$Y_n = T_n T_{n-1} \dots T_1 U \qquad (12)$$

c) Systems with different input wave function and hence with different states of the output, but with the same transfer function T. The relation for the n^{th} processing in this case is:

$$Y_n = T^n U_1 \qquad (13)$$

d) Systems with different input wave function and hence with different states of the output but also with different transfer functions T_i. The relation for the n^{th} processing in this case is:

$$Y_n = T_n U_{n-1} \qquad (14)$$

CONCLUSIONS AND RESULTS

Since almost 100 years ago, when the quantum physics and quantum phenomena have been discovered, many of scientists are questioning themselves and are wondering whether the observed reality is the single available reality or it is merely only a tiny fragment of the reality, namely the fragment that we can perceive due to our own limited senses and the limitations corresponding to our senses.

Other thoughts in this relation are: How could be used the quantum reality phenomena within large scale systems and what are the common aspects shared by the matter at large scale and the matter at quantum level? and What are the implications of quantum phenomena at the large scale? The present paper took into consideration this possible connection and relationship between the phenomena at large scale and the phenomena at quantum level.

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